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THE INNER, INITIAL, MAGNETIC PERMEABILITY OF
IRON AND NICKEL AT ULTRA-HIGH
RADIOFREQUENCIES

A DISSERTATION SUBMITTED TO THE FACULTY
OF THE DIVISION OF THE PHYSICAL SCIENCES
IN CANDIDACY FOR THE DEGREE OF DOCTOR OF
PHILOSOPHY

DEPARTMENT OF PHYSICS

1939

By
JUSTIN L. GLATHART

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The Inner, Initial, Magnetic Permeability of Iron and Nickel at Ultra-High Radiofrequencies

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A new method for the measurement of the ultra-high frequency, initial, inner permeability of ferromagnetic materials has been developed which gives much greater accuracy and is much faster than the older methods. The probable error is under one percent at a frequency of 197 megacycles per second, as contrasted with previous methods in which the error is of the order of 10 to 20 percent. This improvement is due to three factors: (1) the use of a coaxial cable instead of Lecher wires, (2) the use of a fine central wire and (3) correction for the influence of the finite conductivity of the resonance system. The magnetic permeability of high purity, commercial iron wire was found to be 53.8 ± 0.3 at 1.97×10^8 cycles per second, and 25°C . It was constant for all values of tension up to the breaking point of the wire at 980 grams force. It also remained constant for superimposed longitudinal magnetic fields up to 100 oersteds. The permeability of commercially pure nickel was found to be 3.61 ± 0.08 at the same frequency and temperature. As the temperature was raised the permeability increased to a value of 12.22 at 320°C , beyond which it dropped rapidly to unity at 370°C .

INTRODUCTION

THE magnetic properties of ferromagnetic materials at high radiofrequencies have been studied by a number of investigators.¹ The various methods used have proved to be both tedious and inaccurate; the precision in some cases is no better than 10–20 percent. With the object of alleviating these difficulties the method described below has been developed. With this method results accurate to within 0.5 percent can be obtained in a much shorter time.

A survey of the previous work in this field showed considerable discrepancies in the published values of initial permeability of iron and nickel in both the high frequency and the ultra-high frequency ranges. A number of factors might be responsible for these discrepancies, e.g., the purity of the sample, its previous heat and mechanical treatment as well as magnetic history, its tension, temperature, susceptibility to external magnetic fields, etc. The work reported here deals with the effect of external magnetic fields and changes in tension on the initial permeability of commercially pure iron wire, and the effect of temperature on the permeability of commercially pure nickel wire.

Standing waves were set up in a coaxial cable resonating system by means of a frequency and intensity stabilized oscillator operating at approximately 197 megacycles per second. Two similar cable systems were used, in one of which the central conductor was a nonmagnetic wire (copper). In the other cable the central conductor was the magnetic wire (iron or nickel) whose permeability was to be determined. In the case of the latter, the resonance peaks were closer together than those obtained with the nonmagnetic cable. From this wave-length shortening and the known constants of the cable, such as tube and wire radii and electrical conductivity, the permeability of the magnetic sample could be deduced as follows.

THEORY

The coaxial cable has an inductance per unit length given by²

$$L = 2 \log(b/a) + (1/2\pi)[(\rho\mu/\nu a^2)^{\frac{1}{2}} + (\rho'\mu'/\nu b^2)^{\frac{1}{2}}] \text{ e.m.u./cm}, \quad (1)$$

where ν is the frequency of the electrical oscillations, a and b are the radii of the central wire and the outer tube, respectively, of the coaxial cable, ρ and μ are the electrical resistivity and magnetic permeability of the central wire, and ρ' and μ' are the corresponding quantities for the outer tube.

² J. J. Thompson, *Recent Researches in Electricity and Magnetism* (Clarendon Press, Oxford, 1893), p. 295.

¹ (a) W. Arkadiev, *Physik. Zeits.* **22**, 511 (1921); (b) C. Gutton and Mme. I. Mihul, *Comptes rendus* **184**, 1234 (1927); (c) G. R. Wait, F. G. Brickwedde and E. L. Hall, *Phys. Rev.* **32**, 967 (1928); (d) J. B. Hoag and H. Jones, *Phys. Rev.* **42**, 571 (1932); (e) J. Müller, *Zeits. f. Physik* **88**, 143 (1934); (f) G. Potapenko and R. Sängner, *Zeits. f. Physik* **104**, 779 (1937).

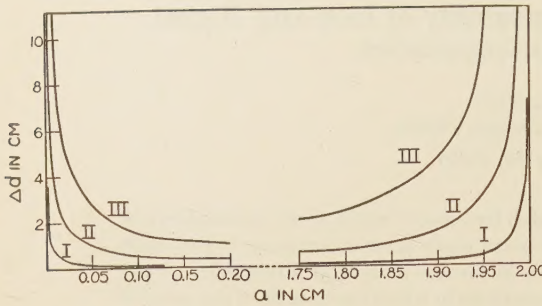


FIG. 1. The wave-length shortening as a function of the radius of the central iron wire of the coaxial cable. I, $d_0 = 25$ cm, $\mu = 23$; II, $d_0 = 76$ cm, $\mu = 53.8$; III, $d_0 = 150$ cm, $\mu = 64$. In all cases $b = 2$ cm, $\rho = 10^4$.

Equation (1) may be written in the form

$$L = L_a + L_i, \quad (2)$$

where $L_a = 2 \log (b/a)$. This depends only on the cable dimensions, while L_i involves the resistivities and permeabilities in addition. From the equation for the velocity of electromagnetic waves in free space, $c = \nu \lambda_0$, and the equation for the velocity of propagation along the cable system, $v = 1/(LC)^{1/2}$, together with the expression for the capacitance of the cable per unit length, $C = \epsilon/[2 \log (b/a)]$, an equation may be derived for the *inner*, initial permeability of the central wire. When the outer tube is of nonmagnetic material, a term in this equation, $(a/b)(\rho'/\rho)^{1/2}$, may be neglected, provided the radius of the central wire, a , is small compared to that of the outer tube, b . In the actual cable used in the present investigation this term amounted to only 0.07 percent of the whole. Introducing the symbols $d_0 \equiv \lambda_0/2$ and $d \equiv \lambda/2$ for the half-wave-lengths in free space and along the cable, respectively, the final equation for the inner permeability may be written³

$$\mu = 1.256 \frac{a^2}{\rho} \left(\log_{10} \frac{b}{a} \right)^2 \frac{(d_0^2 - d^2)^2}{d_0 d^4} \times 10^{13}. \quad (3)$$

OPTIMUM CABLE DIMENSIONS

Since μ depends upon higher powers⁴ of d and $d_0 - d$, not only must the wave-lengths d and d_0

³ The analysis given here follows the same steps as that in the article by Hoag and Jones but is applied to a coaxial rather than a Lecher wire resonance system.

⁴ When d is approximately equal to d_0 Eq. (3) reduces to

$$\mu = \frac{k(d_0 - d)^2(d_0 + d)^2}{d_0 d^4} \doteq k' \frac{(\Delta d)^2}{d^3},$$

where k and k' are constants for a given cable.

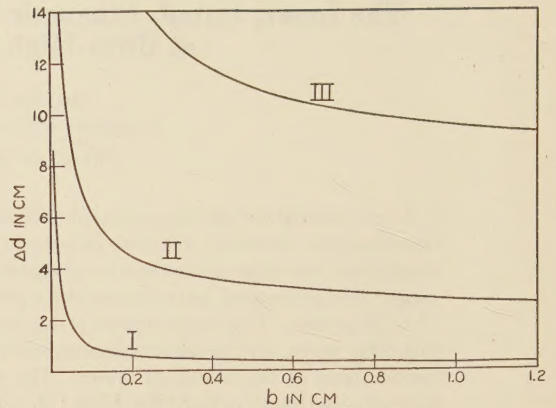


FIG. 2. The wave-length shortening as a function of the radius of the outer tube of the coaxial cable. I, $d_0 = 25$ cm, $\mu = 23$; II, $d_0 = 76$ cm, $\mu = 53.8$; III, $d_0 = 150$ cm, $\mu = 64$. In all cases, $a = 0.01$ cm, $\rho = 10^4$.

be accurately measured but also $d_0 - d \equiv \Delta d$, the wave-length shortening, should be made as large as possible. As shown below, Δd depends upon the cable dimensions and can be made as large as necessary by a proper choice of a and b .

If Eq. (3) is solved for Δd we have

$$\Delta d = d_0 \left\{ 1 - \left[\frac{3.544a \log_{10} (b/a) \times 10^6}{(\mu \rho d_0)^{1/2} + 3.544 \times 10^6 a \log_{10} b/a} \right]^2 \right\}^{1/2}. \quad (4)$$

It can be seen that Δd approaches a maximum equal to d_0 itself as the term in the square brackets approaches zero. If ρ , μ and d_0 are taken as constants, this occurs as $a \rightarrow 0$, $a \rightarrow b$, or $b \rightarrow 0$.

Figure 1 shows Δd as a function of a for an outer tube radius of two cm.⁵ The dependence of Δd upon b , the radius of the outer tube, is shown in Fig. 2. From these curves it can be seen that the best form of resonance cable, from the point of view of large Δd , would consist of a small capillary bore tube having an extremely fine wire stretched along its axis. From a practical standpoint, however, other factors enter the problem, such as the necessity for maintaining adequate insulation between the cable elements, the difficulty of insuring their accurate concentricity, the desirability of low electrical resistance, and the necessity for the introduction of a movable shorting piston.

The cable system finally adopted consisted of

⁵ The values of μ used in computing Δd are the approximate values of Hoag and Jones, reference 1(d), and of Hoag and Gottlieb, Phys. Rev. 55, 410 (1939).

a conveniently large tube, nearly two cm in radius, along the axis of which was stretched a fine wire (radius 0.009 cm) of the material under test. This gave a value for Δd of about three cm in the case of iron, which could be measured with sufficient accuracy. Also such a cable fulfilled the practical requirements mentioned above and justified the neglect of the term $(a/b)(\rho'/\rho)^{\frac{1}{2}}$, which simplified the final equation.

Although, in this investigation, a single wave-length was used throughout, it is of interest to know the manner in which Δd varies with the wave-length. Fig. 3 shows this relationship for several selected values of a , when b remains equal to two cm in all cases. At wave-lengths of several meters the shortening amounts to 30 or 40 cm or more.

DETERMINATION OF THE WAVE-LENGTH OF THE OSCILLATOR

A nonmagnetic coaxial resonance cable was used to obtain the true wave-length of the oscillator. However, the distance between successive resonance peaks is the half-wave-length in free space only when the cable system is of infinite conductivity. This is easily seen if Eq. (4) is written in the form

$$d_0 - d = d_0 \left[1 - \left(\frac{k}{(\rho d_0)^{\frac{1}{2}} + k} \right)^2 \right] \quad (4')$$

where μ has been taken as unity, $k = 3.544 \times 10^6 a \log_{10} (b/a)$, and $d_0 - d$ is the half-wave-length shortening obtained with the nonmagnetic cable. Evidently $d = d_0$, i.e., the cable yields the

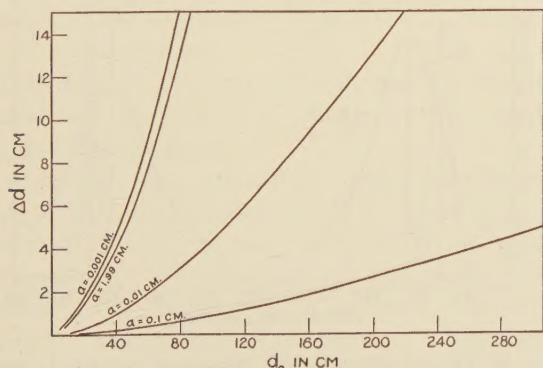


FIG. 3. The wave-length shortening as a function of the wave-length, for iron central wires of various radii. The outer tube radius is 2 cm in all cases.

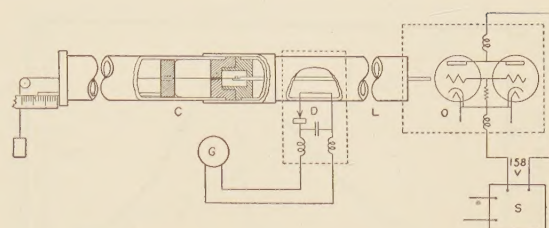


FIG. 4. Diagram of apparatus.

true half-wave-length, only when $\rho = 0$. This resistance shortening is appreciable even for a copper cable. Fortunately, however, Eq. (4') enables its value to be determined. The measured value, d , is related to the true half-wave-length by the equation

$$d = d_0 \left(\frac{k}{(\rho d_0)^{\frac{1}{2}} + k} \right)^2 \quad (5)$$

This can be computed for various values of d_0 and a curve plotted, from which the value of d_0 corresponding to any measured d can be read directly. For the cable described above, the difference between the measured and true values of the half-wave-length amounted to 0.21 cm at $d = 76$ cm.

APPARATUS

The arrangement of the apparatus used is shown in Fig. 4. The ultra-high frequency oscillator is shown at O. S is a voltage stabilizer used to maintain a constant plate voltage. The coaxial resonance cable, C, was coupled inductively to the oscillator by means of a concentric lead L, which contained a longitudinal slot for the introduction of the pick-up wire of the crystal detector circuit D. The latter was connected to a galvanometer whose deflections, plotted as a function of the position of the movable shorting piston in the resonance cable, yielded the desired resonance curves.

For measurements at high temperatures the resonance cable was surrounded along its entire length by a tubular, wire-wound, noninductive, resistance furnace, the temperature of which could be held constant at any desired value during the course of a run.

DISCUSSION

The coaxial resonance system possesses, for this kind of work, a number of distinct advan-

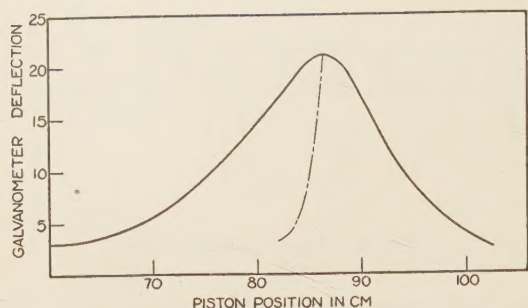


FIG. 5. A single resonance peak showing one type of asymmetry.

tages over an open type of system such as Lecher wires. The most important of these is that, if properly proportioned, it will yield large values of Δd , which enables a much greater accuracy to be attained than has hitherto been possible. The time required to take the necessary data and compute a result has been reduced from two or three hours to 30 or 40 minutes, because of the simple form of the final equation. Since a coaxial line causes less attenuation than an open system the successive resonance peaks do not decrease in height so rapidly nor broaden out so greatly as with an open wire line, thus again facilitating accurate location of the resonance peaks. The closed coaxial system eliminates difficulties caused by coupling between the resonance system and external objects in the room, and so has great stability. Practically perfect reflection of the waves can be attained with a simple form of metallic piston, obviating the necessity for large plate bridges or double bridges. Finally, the sample is used in the form of a single fine wire which is decidedly more convenient than the large rods required by previous methods.⁶

It was necessary that the oscillator be exceedingly stable, both as regards frequency and intensity. This was secured by stabilizing the plate potential by means of a Street and Johnson circuit,⁷ with constants adjusted for the comparatively low voltage and large current required. When properly adjusted this stabilizer circuit held the plate potential constant at 158 volts

even when the line voltage was changed by as much as 15 or 18 volts.

Because the resonance peaks, especially the second ones, were too broad to permit their tips to be located with the desired degree of accuracy, d_0 and d were determined by measuring the distance between the median lines of successive resonance peaks rather than the distance between their tips. This required that each of the peaks be symmetrical about a vertical line through its tip. Laville⁸ has shown, for the similar case of Lecher wires, that this condition of symmetry is also necessary in order to get correct values for wave-length. The distance between the tips of successive peaks, if they are asymmetrical, is not the true half-wave-length. Fig. 5 shows one type of asymmetry encountered. Resonance curves showing asymmetry of this type (left-handed asymmetry) gave high values for d or d_0 , while those having right-handed asymmetry yielded low values, with respect to the values obtained with symmetrical peaks. To attain this condition of symmetry required a careful adjustment of the relative positions of the various parts of the apparatus, especially of the detector circuit along the coaxial lead.

Figure 6 shows two typical resonance curves, each with two symmetrical peaks. Curve I is for a copper (nonmagnetic) coaxial cable and curve II is for one containing an iron central wire. The wave-length shortening is clearly evident. The intermediate portion of the scale of abscissae, where the galvanometer deflections were zero, has been omitted.

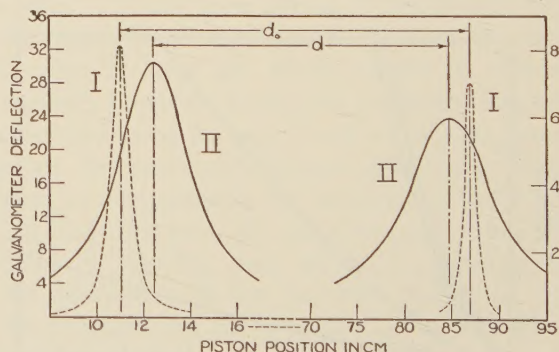


FIG. 6. Typical symmetrical resonance curves. I, non-magnetic coaxial cable; II, cable with iron central wire.

⁶ The method of Potapenko and Sanger, reference 1(j), uses only short wires.

⁷ J. C. Street and T. H. Johnson, *J. Frank. Inst.* **214**, 155 (1932).

⁸ G. Laville, *Ann. de physique* **2**, 328 (1924).

EFFECT OF CHANGES IN TENSION

All data from which resonance curves were plotted were taken with the magnetic central wire under a constant tension of about 50 grams force, which was sufficient to keep the wire taut. In the case of iron wire an investigation was made of the effect of variation in tension upon the permeability. A number of runs were made with tensions ranging from 50 grams force to the breaking point of the wire at about one kilogram. The resonance curves at every value of the tension exactly superposed. No change in μ with tension in the range covered was detected.

EFFECT OF AN EXTERNAL MAGNETIC FIELD

To investigate the effect of an external longitudinal magnetic field upon the permeability of iron wire the resonance cable was completely surrounded by a large solenoid, and fields with intensities ranging from very nearly zero up to 100 oersteds were applied. Here again the resonance curves for all applied field strengths were identical. No change in permeability was noted in the range of field intensities used.

RESULTS

Measurements were made of the permeability of both iron and nickel at room temperature. The iron wire was obtained from a commercial source and was found upon analysis to be 100 percent pure iron within 0.05 percent.

Five separate determinations of the permeability of this wire at room temperature were made at widely different times and with different samples. The average of these with its probable error is given below.

For iron at 25°C, at a frequency of 1.97×10^8 cycles/sec. $\mu = 53.8 \pm 0.3$. This is an uncertainty of about 0.5 percent.

Similarly, the ultra-high frequency permeability of a high purity nickel obtained from a commercial source was determined at room temperature. The analysis of the nickel was found to be: nickel, 99.27 percent; iron, 0.14 percent; manganese, 0.10 percent; chromium, 0.0 percent; and cobalt, trace. In all, seven determinations of the permeability were made, on several different days and with different

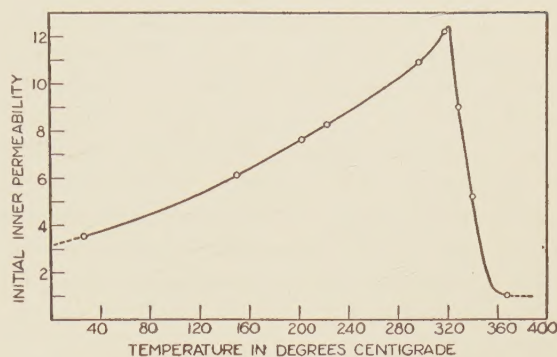


FIG. 7. Inner, initial permeability of nickel, at a frequency of 1.97×10^8 cycles per sec., plotted as a function of temperature.

samples of this wire, which yielded as a mean $\mu = 3.61 \pm 0.08$ for nickel at 27°C and a frequency of 1.97×10^8 cycles/sec. This is an uncertainty of 2.2 percent.

PERMEABILITY OF NICKEL AS A FUNCTION OF TEMPERATURE

The nickel wire was subjected to temperatures ranging from 27°C to 368°C, the latter of which is beyond the Curie point for nickel, and the permeability measured at these and intermediate temperatures. The initial permeability as a function of temperature is shown by the curve of Fig. 7.

The points on this curve represent readings taken on two successive days with different samples of the nickel wire. The wave-length of the oscillator was measured several times during the course of these readings and although it was practically constant, each nickel resonance curve was compared with its corresponding copper curve. By assuming the slight variations in wave-length to be due entirely to experimental error in measurement, the wave-length λ_0 was found to be equal to 151.89 ± 0.04 cm. This is an uncertainty of 0.026 percent.

As may be seen from Eq. (3), the resistivity of the nickel wire had to be known for every temperature at which μ was to be determined. The resistivity vs. temperature curve was determined by a separate experiment. The wire was put in place in the cable and all conditions were made similar to those under which μ was measured.

A rise in temperature would also affect the radii a and b of the wire and tube. The maximum values of these expansion corrections were computed and their effects found to be negligible. The amount of expansion of the rod controlling the movements of the piston could be computed from the known temperatures, the known amount of rod exposed and its coefficient of expansion. All scale readings were individually corrected for this expansion.

VELOCITY OF THE WAVES ALONG THE CABLE

From the accepted value for the velocity of the electromagnetic waves in free space, $c=2.99775 \times 10^{10}$ cm/sec., and the true wave-length of the oscillator, $\lambda_0=151.89$ cm, the frequency of the latter was found to be 1.9736×10^8 cycles/sec.

This combined with the measured wave-lengths along the cables containing copper, nickel and iron wires gave for the velocities of wave propagation along these cables at room temperature the following results. The velocity along the copper wire cable proved to be 2.993×10^{10} cm/sec., that along the nickel wire cable was 2.964×10^{10} cm/sec., and that along the iron wire cable was 2.936×10^{10} cm/sec. These differ from the wave-length in free space by 0.17 percent, 1.13 percent and 2.06 percent, respectively.

I wish to express my sincere appreciation to Professor J. Barton Hoag for suggesting this problem and for his encouragement and advice during the progress of the investigation. For the chemical analysis of the iron and nickel wire I am indebted to Mr. Alfred Klapperich.

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